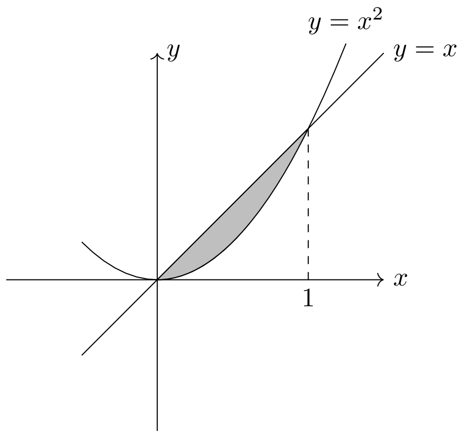
MATH 2E Prep: Double Integrals

1. Evaluate the double integral $\iint_D 6(x+y)^2 dS$, where D is the region between the graphs of $y = x$ and $y = x^2$.

Solution:



From the figure above, D can be expressed as $D = \{(x, y) \mid 0 \le x \le 1,\ x^2 \le y \le x\}$. So

$$\begin{aligned}
\iint_D 6(x+y)^2 dS &= \int_0^1 \int_{x^2}^{x} 6(x+y)^2 dy dx \\
&= \int_0^1 \left[2(x+y)^3\right]_{x^2}^{x} dx \\
&= \int_0^1 \left[16x^3 - 2(x+x^2)^3\right] dx \\
&= \int_0^1 -2x^6 - 6x^5 - 6x^4 + 14x^3 dx \\
&= -\frac{2x^7}{7} - x^6 - \frac{6}{5}x^5 + \frac{7x^4}{2}\Big]_0^1 \\
&= -\frac{2}{7} - 1 - \frac{6}{5} + \frac{7}{2} = \frac{71}{70}.
\end{aligned}$$

2. Evaluate the double integral $\int_0^1 \int_{\sqrt{x}}^{1} \sqrt{y^3+1}\, dy dx$ by reversing the order of integration.

Solution: Let D be the region described by

$$D = \{(x, y) \mid 0 \le x \le 1,\ \sqrt{x} \le y \le 1\},$$

A figure for D is provided at the end of this solution. By the figure, D can also be described as

$$D = \{(x, y) \mid 0 \le y \le 1,\ 0 \le x \le y^2\},$$

Therefore,

$$\begin{aligned}
\int_0^1 \int_{\sqrt{x}}^1 \sqrt{y^3+1}\,dydx &= \iint_D \sqrt{y^3+1}\,dS \\
&= \int_0^1 \int_0^{y^2} \sqrt{y^3+1}\,dxdy \\
&= \int_0^1 \left[x\sqrt{y^3+1}\right]_0^{y^2} dy \\
&= \int_0^1 y^2\sqrt{y^3+1}\,dy \\
&= \int_0^1 \frac{1}{3}\sqrt{u+1}\,du \qquad \text{(where } u = y^3\text{)} \\
&= \left[\frac{2}{9}(u+1)^{\frac{3}{2}}\right]_0^1 \\
&= \frac{4\sqrt{2}-2}{9}
\end{aligned}$$

Figure for D: